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# **Phenomenological Quark Mass Matrix Model with Two Adjustable Parameters**

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## **Abstract**

A phenomenological quark mass matrix model which includes only two adjustable parameters is proposed from the point of view of the unification of quark and lepton mass matrices. The model can provide reasonable values of quark mass ratios and Kobayashi-Maskawa matrix parameters.

It is widely accepted that the family number of ordinary quarks and leptons is three. (This does not ruled out a possibility that there are some extraordinary families, e.g. a family with an extremely heavy neutrino, and so on.) Then, we have ten observable quantities related to up- and down-quark mass matrices,  $M_u$  and  $M_d$ , i.e., six up- and down-quark masses and four parameters of Kobayashi-Maskawa (KM) [1] matrix. On the other hand, most of quark mass matrix models currently proposed include adjustable parameters more than five (two parameters for each quark mass matrix  $M_q$  ( $q = u, d$ ) and one relative phase parameter between up- and down-quark mass matrix phase parameters). At present, every model is comparably plausible, and is in agreement with the present experimental data. Nevertheless, we cannot resist the temptation to investigate a further new-type mass matrix form of  $(M_u, M_d)$  with parameters less than four, because we expect that the quark and lepton families are governed by a more fundamental law of the nature.

In the present paper, we propose the following model of quark and lepton mass matrices inspired by an extended technicolor-like model:

$$M_f = m_0 G O_f G , \quad (1)$$

$$G = \text{diag}(g_1, g_2, g_3) , \quad (2)$$

$$O_f = \mathbf{1} + 3a_f X(\phi_f) , \quad (3)$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad X(\phi) = \frac{1}{3} \begin{pmatrix} 1 & e^{i\phi} & 1 \\ e^{-i\phi} & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \quad (4)$$

where  $f = \nu, e, u$ , and  $d$  are indices for neutrinos, charged leptons, up- and down-quarks, respectively. Here, the diagonal matrix  $G$  denotes a coupling constant matrix of a hypercolored boson  $\phi_\alpha$  with ordinary fermions  $f_i$  and hypercolored fermions  $F_{i\alpha}$  ( $\alpha$  and  $i$  are hypercolor and family indices, respectively), and the matrix  $O_f$  denotes the condensation of the hypercolored fermions  $\langle(\bar{F}F)\rangle$ . Since we consider the so-called seesaw mechanism [2] for neutrino mass matrix, the matrix  $M_\nu$  given in (1) should be taken as the Dirac mass matrix part of the neutrino mass matrix.

As we discuss below, since we take  $a_e = 0$  in the charged lepton mass matrix  $M_e$ , the parameters  $g_i$  are fixed as  $\sqrt{m_0}G = \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ , so that the

mass matrix  $M_f$  is effectively given by

$$M_f = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} + a_f \begin{pmatrix} m_e & e^{i\phi_f} \sqrt{m_e m_\mu} & \sqrt{m_e m_\tau} \\ e^{-i\phi_f} \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_\mu m_\tau} \\ \sqrt{m_e m_\tau} & \sqrt{m_\mu m_\tau} & m_\tau \end{pmatrix}. \quad (5)$$

In the present paper, we put an ansatz for  $\phi_f$ , ( $\phi_u = 0$ ,  $\phi_d = \pi/2$ ), so that adjustable parameters in the quark mass matrices  $M_u$  and  $M_d$  are only two,  $a_u$  and  $a_d$ . As we demonstrate later, a suitable choice of the parameters  $a_u$  and  $a_d$  will provide not only reasonable values of up- and down-quark mass ratios  $m_i^u/m_j^u$  and  $m_i^d/m_j^d$  ( $i, j = 1, 2, 3$ ), respectively, but also reasonable values of the ratios  $m_i^u/m_i^d$  as well as reasonable values of KM matrix parameters.

The mass matrix forms  $M_e$  and  $M_\nu$  have already proposed by the author [3,4] from the phenomenological point of view. In fact, the present quark mass matrix model was inspired by the phenomenological success of the charged and neutrino mass matrices as we review below.

Ten years ago, the author [3] has proposed a charged lepton mass matrix model, in which charged lepton masses  $m_i^e = (m_e, m_\mu, m_\tau)$  are generated through the condensations of hypercolored fermions  $E_{i\alpha}$ ,  $\langle(\bar{E}E)\rangle$ , and the exchanges of a hypercolored vector boson  $\phi_\alpha$  which is coupled with  $\sum_i g_i \bar{e}_i E_{i\alpha}$ , i.e., the masses  $m_i^e$  are given by  $m_i^e \simeq g_i^2 \langle(\bar{E}E)\rangle / m_\phi^2$ . (The model is similar to the extended technicolor model [5], but we consider that the vector boson  $\phi_\alpha$  is not a gauge boson.) Here, the hypercolored boson  $\phi_\alpha$  (hereafter we drop the index  $\alpha$ ) is a particularly mixed state among  $SU(3)$ -family octet bosons  $\phi_3$  and  $\phi_8$  and singlet boson  $\phi_0$ , which are the  $\lambda_3$ ,  $\lambda_8$  and  $\lambda_0$  components of  $SU(3)$ . We consider that the octet bosons acquire large masses at an energy scale  $\Lambda_H$  except for one component  $\phi^{(8)}$  which is a linear combination of  $\phi_3$  and  $\phi_8$ , while  $\phi^{(8)}$  has exactly the same mass as  $\phi_0$ . Then, if there is a mixing term between  $\phi^{(8)}$  and  $\phi_0$ , the  $45^\circ$  mixing between  $\phi^{(8)}$  and  $\phi_0$  is inevitably caused. We assume that only one of the two states can contribute to the mass matrix  $M_e$ , so that the coupling constant  $g_i$  is given by  $g_i = (g_i^{(8)} + g_0)/\sqrt{2}$ , where  $g_i^{(8)}$  and  $g_0$  are coupling constants of  $\phi^{(8)}$  and  $\phi_0$  with  $\bar{e}_i E_i$ , respectively, and they satisfy the relations  $g_1^{(8)} + g_2^{(8)} + g_3^{(8)} = 0$  and  $(g_1^{(8)})^2 + (g_2^{(8)})^2 + (g_3^{(8)})^2 = 3g_0^2$ , because we consider that  $\phi^{(8)}$  and  $\phi_0$  belong to the nonet of  $U(3)$ -family. Then,

the coupling constants  $g_i$  satisfy the relation

$$g_1^2 + g_2^2 + g_3^2 = \sum_{i=1}^3 \left( \frac{g_i^{(8)} + g_0}{\sqrt{2}} \right)^2 = 3g_0^2 = \frac{2}{3} \left( \sum_{i=1}^3 \frac{g_i^{(8)} + g_0}{\sqrt{2}} \right)^2 = \frac{2}{3} (g_1 + g_2 + g_3)^2, \quad (6)$$

which leads to a charged lepton mass sum rule [3]

$$m_e + m_\mu + m_\tau = \frac{2}{3} (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2. \quad (7)$$

The sum rule (7) predicts  $m_\tau = 1776.97$  MeV from the input values of  $m_e$  and  $m_\mu$ . The predicted value 1777 MeV is in excellent agreement with the observed values of  $m_\tau$  which have recently been reported by ARGUS [6], BES [7] and CLEO [8] collaborations. Thus, the phenomenological success of the charged lepton mass matrix  $M_e = m_0 G \mathbf{1} G$  is our main motivation to consider the mass matrix form of the type  $m_0 G O_f G$ .

In Ref. [3], the boson state  $\phi$  is more explicitly given by

$$\phi = -\frac{1}{\sqrt{2}} \left[ \cos\left(\frac{\pi}{4} - \epsilon\right) \phi_3 - \sin\left(\frac{\pi}{4} - \epsilon\right) \phi_8 \right] + \frac{1}{\sqrt{2}} \phi_0. \quad (8)$$

As a result, the matrix  $G$  is given by

$$G = \frac{1 + \epsilon}{2\sqrt{2}\sqrt{1 + \epsilon^2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1 - \epsilon}{2\sqrt{6}\sqrt{1 + \epsilon^2}} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

where  $\cos(\pi/4 - \epsilon)$  and  $\sin(\pi/4 - \epsilon)$  are replaced by  $(1 + \epsilon)/\sqrt{2(1 + \epsilon^2)}$  and  $(1 - \epsilon)/\sqrt{2(1 + \epsilon^2)}$ , respectively. In the limit of “ideal mixing”, i.e.,  $\epsilon = 0$ , the model leads to massless electron. This explains why electron mass is extremely small compared with other charged lepton masses.

The motivation to consider the matrix form  $O_f$  of the type  $\mathbf{1} + 3a_f X$  is as follows: Recently, in order to explain a neutrino mixing value  $\sin \theta_{e\mu} \simeq 0.04$  ( $\sin^2 2\theta_{e\mu} \simeq 7 \times 10^{-3}$ ) suggested by GALLEX [9], the author [4] has proposed a neutrino mass matrix model, in which the neutrino mass matrix  $M_\nu$  is given by  $M_\nu \simeq M_\nu^D M_M^{-1} M_\nu^D = (M_\nu^D)^2 / m_M$  ( $m_M$  is a Majorana neutrino mass) on the basis of the conventional seesaw mechanism scenario [2], and the Dirac mass matrix  $M_\nu^D$  is given by the form  $M_\nu^D = m_0^\nu G(\mathbf{1} + 3a_\nu X(0))G$ , where  $a_\nu$  is a numerical parameter with  $a_\nu \gg 1$ . Here we have supposed that the hypercolored neutrino condensation  $\langle(\overline{N}N)\rangle$  takes a democratic term ( $X(0)$ -term) dominance form,  $\mathbf{1} + 3a_\nu X(0)$ , differently from the case of  $\langle(\overline{E}E)\rangle \propto \mathbf{1}$ . The model can lead to a desirable prediction [4]  $\sin \theta_{e\mu} \simeq (1/2)\sqrt{m_e/m_\mu} \simeq 0.035$  for  $a_\nu \gg 1$ .

In general, the mass matrix (5) with  $\phi_f = 0$  provides the relation [4]

$$m_1^f/m_2^f \simeq 3m_e/4m_\mu = 0.00363, \quad (10)$$

in the limit of  $1/a_f \rightarrow 0$ , where  $m_i$  are eigenvalues of the mass matrix (5) and are defined as  $|m_1| < |m_2| < |m_3|$ . The conventional values [10] of the running quark masses at 1 GeV,  $m_u \simeq 5.1$  MeV and  $m_c \simeq 1.35$  GeV, provide  $m_u/m_c \simeq 0.0038$ , which is in agreement with the prediction (10). This is just the motivation to consider the mass matrix  $M_u$  given by (1) with  $\phi_u = 0$ , i.e.,

$$M_u = m_0^u G(\mathbf{1} + 3a_u X(0))G. \quad (11)$$

Hereafter, for convenience, we will refer the Gasser–Leutwyler’s values [10] for  $\Lambda_{\overline{MS}}^{(3)} = 0.150$  GeV [11] as running quark mass values (in unit of GeV) at an energy scale 1 GeV:

$$\begin{aligned} m_u &= 0.0051 \pm 0.0015, & m_c &= 1.35 \pm 0.05, & m_t &= 226_{-49}^{+43}, \\ m_d &= 0.0089 \pm 0.0026, & m_s &= 0.175 \pm 0.055, & m_b &= 5.58 \pm 0.13. \end{aligned} \quad (12)$$

The value of  $m_t(1 \text{ GeV})$ , which is not listed in the original paper by Gasser and Leutwyler, has been estimated by using the standard model parameter fitting value  $m_t^{phys} = 130_{-28}^{+25}$  GeV [12] and  $\Lambda_{\overline{MS}}^{(3)} = 0.150$  GeV ( $\Lambda_{\overline{MS}}^{(4)} = 0.114$  GeV,  $\Lambda_{\overline{MS}}^{(5)} = 0.0699$  GeV). However, the values in (12) should not be taken rigidly, because the estimates are highly dependent on the value of  $\Lambda_{\overline{MS}}$  and models (prescriptions) at present [13].

The mass matrix given by (11) actually can predict reasonable up-quark mass ratios: for example,  $m_u/m_c = 0.00389$  (0.00379) and  $m_c/m_t = 0.00597$  ( $-0.00598$ ) for  $a_u = 16.45$  ( $a_u = -19.02$ ). Here, since the quark mass ratio  $m_u/m_c$  is insensitive to the value of  $a_u$ , we have determined the value of  $a_u$  from the value of  $m_c/m_t$  in (12). The prediction of  $m_u/m_c$  is in excellent agreement with the value of  $m_u/m_c$  provided by (12).

Next, we seek for a mass matrix form for down-quarks. We cannot choose the same mass matrix form as that for up-quarks, i.e.,  $M_d = m_0^d G(\mathbf{1} + 3a_d X(0))G$ , because it leads to a wrong down-quark mass ratio  $m_1^d/m_2^d \simeq 3m_e/4m_\mu$ . Besides, we must introduce a  $CP$  violation phase to the model. We assume a down-quark matrix form  $M_d$  which is similar to  $M_u$ , but which has a phase factor  $\phi_d \neq 0$  as given by (4).

In general, the eigenvalues  $m_i^f$  of the mass matrix (5) are given by

$$\frac{m_1^f}{m_\tau} \simeq \frac{\kappa_f(3 + \kappa_f) - 4\sin^2(\phi_f/2)}{\kappa_f^2(2 + \kappa_f)} \varepsilon_1, \quad \frac{m_2^f}{m_\tau} \simeq \frac{2 + \kappa_f}{1 + \kappa_f} \varepsilon_2, \quad \frac{m_3^f}{m_\tau} \simeq \frac{1 + \kappa_f}{\kappa_f}, \quad (13)$$

where  $\kappa_f = 1/a_f$ ,  $\varepsilon_1 = m_e/m_\tau$  and  $\varepsilon_2 = m_\mu/m_\tau$ . Note that, differently from the case of  $M_u$  with  $\phi_u = 0$ , we cannot take a limit of  $\kappa_d \rightarrow 0$  in the down-quark mass matrix  $M_d$ , because the mass ratio  $m_1^d/m_2^d$  includes a factor  $1/\kappa_d^2$ . The relation

$$\frac{m_s}{m_b} \simeq \frac{(2 + \kappa_d)\kappa_d}{1 + \kappa_d} \frac{m_\mu}{m_\tau} \quad (14)$$

suggests a small but visible value  $\kappa_d \simeq -0.2$  because  $|m_s/m_b| \simeq 0.03$  and  $m_\mu/m_\tau \simeq 0.06$ . Then, the relation

$$\frac{m_d m_s}{m_b^2} \simeq -\frac{4}{(1 + \kappa_d)^3} \frac{m_e m_\mu}{m_\tau^2} \sin^2 \frac{\phi_d}{2} \quad (15)$$

suggests  $|\phi_d| \simeq \pi/2$ . For simplicity, we fix  $\phi_d$  to be  $\phi_d = \pi/2$ , which leads to a maximal  $CP$  violation.

In conclusion, we assume that the down-quark mass matrix  $M_d$  is given by

$$M_d = m_0^d G \left( \mathbf{1} + 3a_d X\left(\frac{\pi}{2}\right) \right) G. \quad (16)$$

Then, a suitable choice of  $a_d$  can provide excellent predictions of  $m_d/m_s$  and  $m_s/m_b$ : for example,  $m_d/m_s = -0.0507$  and  $m_s/m_b = -0.0313$  for  $a_d = -4.81$ . It is noted that, in the mass matrix  $M_f$  with  $\phi_f = \pi/2$ , in general, two values of  $a_f$ ,  $a_f = a_f^{(1)}$  and  $a_f = a_f^{(2)}$ , which satisfy the relation  $(1/a_f^{(1)}) + (1/a_f^{(2)}) = -2$ , can yield the same mass ratios  $m_1^f/m_2^f$  and  $m_2^f/m_3^f$ . Therefore, the alternative choice  $a_d = a_d^{(2)} = -0.558$  provides the same predictions of the down-quark mass ratios as the case of  $a_d = a_d^{(1)} = -4.81$ .

The quark mass matrix model  $(M_u, M_d)$  given in (11) and (16) predicts the KM matrix elements  $V_{ij}$  in the limit of  $1 \ll |\kappa_d| \ll |\kappa_u| \rightarrow 0$  as follows:

$$|V_{us}|^2 \simeq 2 \frac{1 + \kappa_d}{(2 + \kappa_d)^2 \kappa_d^2} \frac{m_e}{m_\mu}, \quad (17)$$

$$|V_{cb}|^2 \simeq \frac{\kappa_d^2}{(1 + \kappa_d)^2} \frac{m_\mu}{m_\tau} \simeq \frac{m_e/m_\tau}{|V_{us}|^2}, \quad (18)$$

$$|V_{ub}|^2 \simeq \frac{m_e}{m_\tau}. \quad (19)$$

The relation (17) leads to the well-known Weinberg–Fritzsch empirical relation [14]  $|V_{us}| \simeq \sqrt{-m_d/m_s}$ , because the mass ratio  $m_d/m_s$  is given by

$$\frac{m_d}{m_s} \simeq -\frac{(1 + \kappa_d)^2 (2 - 2\kappa_d - \kappa_d^2)}{(2 + \kappa_d)^2 \kappa_d^2} \frac{m_e}{m_\mu}. \quad (20)$$

The predicted values of  $|V_{cb}|$  and  $|V_{ub}|$  from (18) and (19) are somewhat large compared with the observed values. This disagreement comes from the approximation in which we took  $\kappa_u = 0$ . The values of  $|V_{ij}|$  are sensitive to  $\kappa_u$  and  $\kappa_d$  as well as to  $\varepsilon_1$  and  $\varepsilon_2$ . As we demonstrate below, a suitable choice of  $\kappa_u = 1/a_u$  can predict reasonable values of  $|V_{cb}|$  and  $|V_{ub}|$  numerically.

In Table I, we show predictions on the KM matrix parameters for the values of  $a_u$  and  $a_d$  which provide reasonable quark mass ratios. We also list the prediction of the rephasing invariant quantity  $J$  [15]. The case  $a_d = a_d^{(1)} = -4.81$  can provide reasonable values of the KM matrix parameters except that the value of  $|V_{ub}|$  is somewhat small. The value of  $|V_{ub}|$  is highly sensitive to the value of the phase parameter  $\phi_d$  when  $M_d$  is given by  $M_d = m_0^d G(\mathbf{1} + 3a_d X(\phi_d))G$ , and a choice of

$\phi_d$  slightly different from  $\phi_d = \pi/2$  predicts a fairly large value of  $|V_{ub}|$  compared with the case of exact  $\phi_d = \pi/2$ . It is likely that the prediction of  $|V_{ub}|$  becomes reasonable value by renormalization effects for  $M_q$ .

On the other hand, the second case  $a_d = a_d^{(2)} = -0.558$  cannot provide reasonable values of  $|V_{cb}|$  and  $|V_{ub}|$  as seen in Table I. However, it should be noted that the case  $a_d = -0.558$  can provide not only the excellent predictions of  $m_d/m_s$  and  $m_s/m_b$  but also the excellent prediction of  $m_d/m_u$  if we consider  $m_0^u = m_0^d$ . When we put  $m_2^d = 0.175$  GeV (i.e.,  $m_0^u/m_0^e = m_0^d/m_0^e = 6.52$ ) in order to compare our prediction with the Gasser–Leutwyler’s values (12), we obtain the following quark mass values at energy scale 1 GeV for the case of  $(a_u, a_d) = (-19.02, -0.558)$ :

$$\begin{aligned} m_1^u &= 0.00504 \text{ GeV} , & m_2^u &= +1.33 \text{ GeV} , & m_3^u &= -223 \text{ GeV} , \\ m_1^d &= 0.00887 \text{ GeV} , & m_2^d &= -0.175 \text{ GeV} , & m_3^d &= +5.59 \text{ GeV} . \end{aligned} \quad (21)$$

The values (21) are in excellent agreement with the Gasser–Leutwyler’s values (12). In most of the conventional quark mass matrix models, if we want to explain the fact  $m_t \gg m_b$ , then we must be contented with saying that the fact  $m_u \sim m_d$  is an accidental coincidence in the model. In the case of  $a_d = a_d^{(2)}$ , we can obtain the reasonable ratio of  $m_u/m_d$  together with the reasonable ratios  $m_i^u/m_j^u$  and  $m_i^d/m_j^d$ . Therefore, the case of  $a_d = -0.558$  is worth being taken into consideration as well as the case  $a_d = -4.81$ .

It should be also be noted that predictions of  $|V_{ij}|$  in the case of  $a_d^{(2)}$  are, in general, exactly the same as those in the case of  $a_d^{(1)}$  if we take

$$V = U_u P U_d^\dagger , \quad (22)$$

$$P = \text{diag}(1, 1, -1) , \quad (23)$$

instead of  $V = U_u U_d^\dagger$ . The modification (22) means that the mass matrices  $(M_u; M_d)$  given by (11) and (16) are not those for the weak eigenstate quark basis  $(u_0, c_0, t_0; d_0, s_0, b_0)$ , but those for the quark basis  $(u_0, c_0, \pm t_0; d_0, s_0, \mp b_0)$ . Although the origin of such phase inversion is not clear, if we accept the scenario, we can provide not only the reasonable values (21) of quark masses but also reasonable values of the KM matrix parameters

$$\begin{aligned} |V_{us}| &= 0.203 , & |V_{cb}| &= 0.0393 , & |V_{ub}| &= 0.00139 , & |V_{td}| &= 0.00882 , \\ J &= 0.891 \times 10^{-5} , \end{aligned} \quad (24)$$



by fixing  $(a_u, a_d) = (-19.02, -0.558)$ .

So far, we have neglected the energy scale dependence of the quark masses and KM parameters. We consider that the mass matrix form (1) is given at an energy scale  $\mu = M_X$ . We expect that fine tuning of our parameters in consideration of the renormalization group equations can provide further excellent predictions of quark masses and KM mixing parameters.

We consider that  $m_0^q$  and  $m_0^e$  satisfy  $m_0^u = m_0^d = m_0^e$  at the energy scale  $M_X$  and the value  $(m_0^q/m_0^e)_{1\text{GeV}} = 6.52$  will be explained by evolving  $m_0^q$  and  $m_0^e$  down from  $M_X$  to 1 GeV. In the present model, the energy scale  $M_X$  need not be identical with the weak boson mass scale  $v \simeq 250$  GeV. In order to give rough estimate of  $M_X$ , we neglect electroweak interaction and use, for convenience, the equation for QCD running quark mass (for example, see Ref. [10]) (not the renormalization group equation for the Yukawa couplings). The value of  $M_X$  estimated is highly sensitive to the choice of  $\Lambda_{QCD}$  ( $\Lambda_{\overline{MS}}^{(n)}$ ). If we adopt a recent experimental value  $\Lambda_{\overline{MS}}^{(4)} = 0.260$  GeV [16] ( $\Lambda_{\overline{MS}}^{(3)} = 0.311$  GeV,  $\Lambda_{\overline{MS}}^{(5)} = 0.175$  GeV,  $\Lambda_{\overline{MS}}^{(6)} = 0.0709$  GeV), we obtain  $M_X \sim 10^{18}$  GeV. The value of  $M_X$  is somewhat large. However, the present estimate of  $M_X$  is only a trial and it should not be taken seriously. The estimate is also highly dependent on the models. In order to give more accurate estimate of  $M_X$ , we must build the model more concretely.

In conclusion, we have proposed a phenomenological quark and lepton mass matrix model (1). The matrix form (1) has a possibility of unified description of quark and lepton masses and their mixings. The mass matrix form  $m_0 G O_f G$  can be understood from an extended technicolor-like scenario (but our boson  $\phi_\alpha$  is not a gauge boson). However, such a mass matrix form (1) can also be understood from a Higgs-boson scenario with some additional  $U(1)$  charges. In both scenarios, it is essential that there are heavy fermions which behave as intermediate states in the mass generation mechanism of the light fermions. In the derivation of the sum rule (7), it is essential that the  $45^\circ$  mixing between octet and singlet parts in the  $U(3)$ -family nonet scheme. In Ref. [17], the sum rule (7) has been re-derived from a Higgs potential model with a mixing term between  $SU(3)$ -family octet and singlet. However, Ref. [17] did not discuss clearly on the additional  $U(1)$  charges which should be introduced in the scenario. Recently, a detailed study of the  $U(1)$  charges related to the horizontal symmetry has been given by Leurer, Nir and Seiberg [18]. We will find a clue to the justification of the present scenario in their paper, in which we can see relations of  $|V_{ij}|$  similar to our relations (17)–(19),

although in our model the parameters  $\kappa_u$  and  $\kappa_d$  are not negligible. However, the purpose of the present paper is to propose a new-type mass matrix form (1), and not to give a reasonable mass generation mechanism for the mass matrix form (1). Theoretical justification of the model (1) will be given elsewhere.

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Table I. Prediction on the KM matrix parameters.

$a_u$	$a_d$	$ V_{us} $	$ V_{cb} $	$ V_{ub} $	$ V_{td} $	$ V_{ub}/V_{cb} $	$J$
+16.45	−4.81	0.204	0.0624	0.00186	0.01291	0.0298	$2.31 \times 10^{-5}$
−19.02	−4.81	0.203	0.0393	0.00139	0.00882	0.0353	$0.891 \times 10^{-5}$
+16.45	−0.558	0.201	0.495	0.0170	0.0907	0.0344	$1.10 \times 10^{-3}$
−19.02	−0.558	0.199	0.515	0.0169	0.0942	0.0328	$1.12 \times 10^{-3}$